# Varibility in aftershock productivity, and its relationship with stress drop

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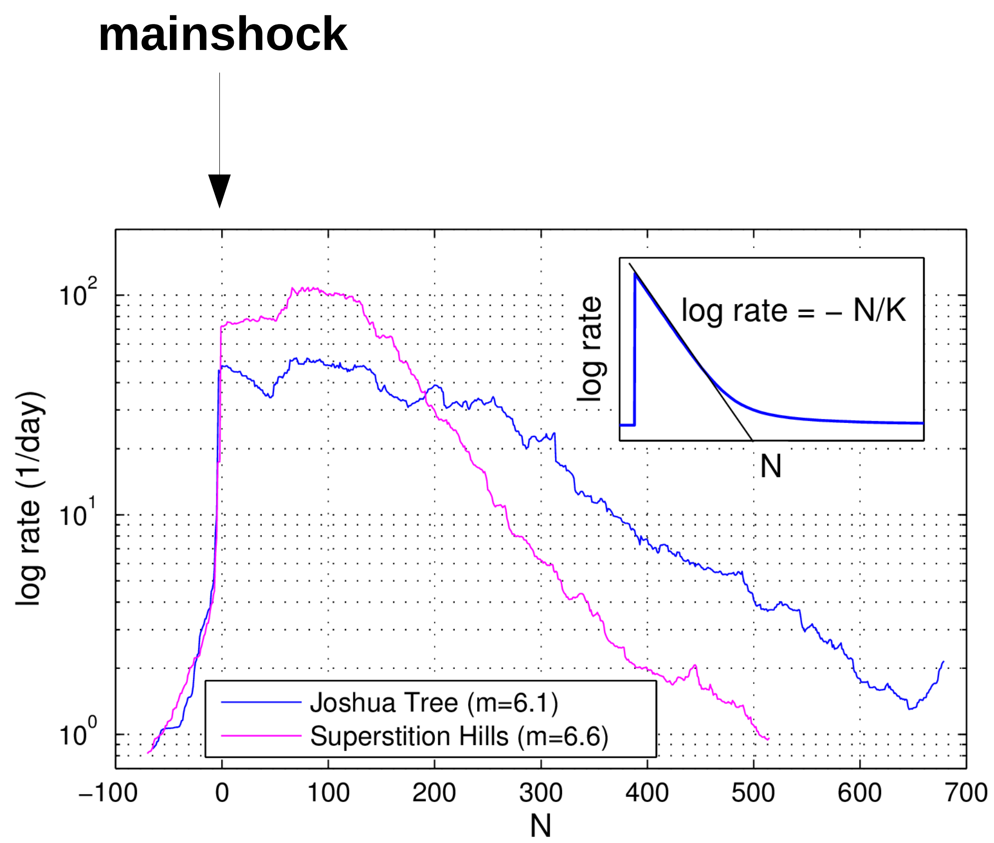
Number of aftershocks $n = K e^{\alpha m} = K\, 10^{a m}$

a-value typically range from 0.6 (MLE approaches) to 1 (space-time window techniques)

**Q : how does triggering departs from this mean law, from one mainshock to the other ?**

↕

**Q : variability of pre-factor K ?**

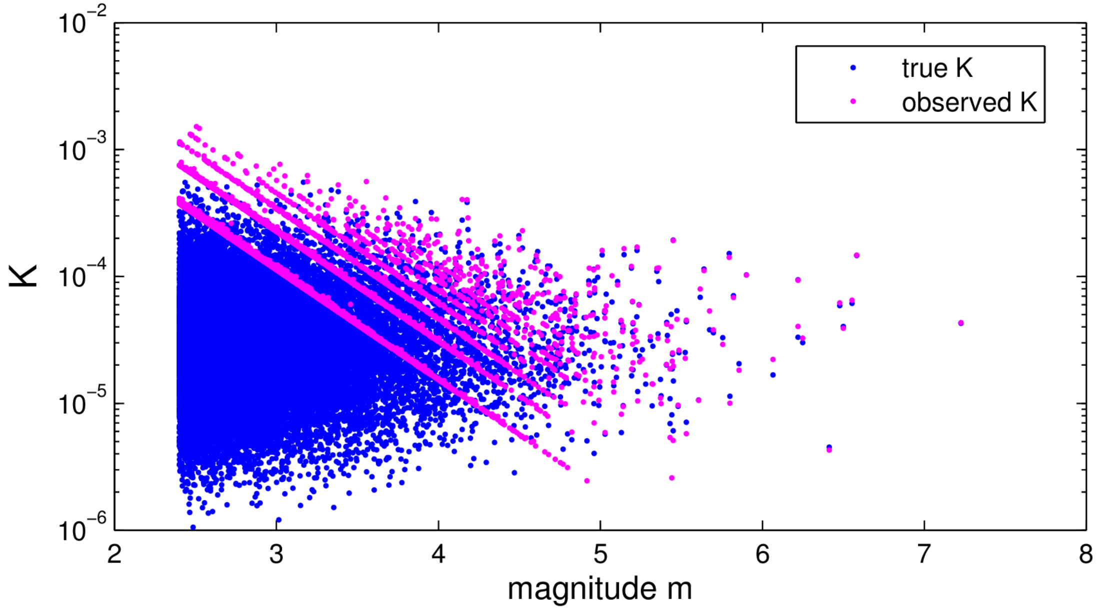



- Smoothed rate of earthquakes vs. number N of earthquakes
- M ≥ 2.4 earthquakes
- 2 months following the 1992 M6.1 Joshua Tree earthquake and the 1987 M6.2 Elmore Ranch and M6.6 Superstition Hill doublet

$$n_i = Poisson\left(K_i e^{\alpha m_i}\right) \quad \text{with} \quad E\{K\} = \mu_K, var(K) = \sigma_K^2$$

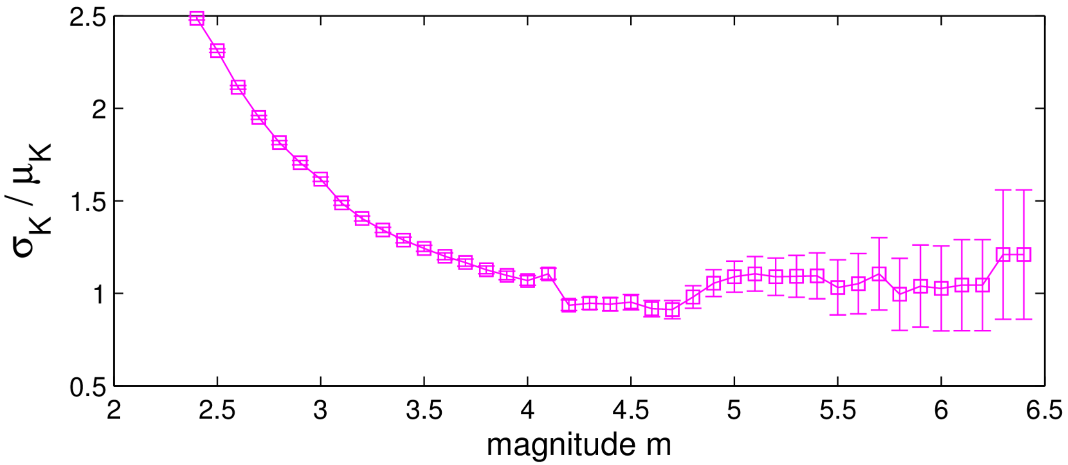
**GOAL is to estimate the variability of K :** $\dfrac{\sigma_K}{\mu_K}$

$$n_i = Poisson\left(K_i e^{\alpha m_i}\right) \Rightarrow \text{Observed} \quad \hat{K}_i = n_i e^{-\alpha m_i}$$



*For a synthetic ETAS catalog with $\sigma_K$ / $\mu_K$ = 1, true parents / daughters are known.*

$$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}}$$ for earthquakes with magnitude ≥ m



$$n_i = Poisson\left(K_i e^{\alpha m_i}\right) \quad \text{with} \quad E\{K\} = \mu_K, var(K) = \sigma_K^2$$

**If $n_i$ and α are estimated correctly :**

Observed $\hat{K}_i = n_i e^{-\alpha m_i}$

$$E\{\hat{K}_i\} = e^{-\alpha m_i} E\{Poisson\left(K_i e^{\alpha m_i}\right)\} = E\{K_i\} = \mu_K$$

$$E\{\hat{K}_i^2\} = \mu_K e^{-\alpha m_i} + E\{K^2\} \Rightarrow \frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}} = \sqrt{\left(\frac{\sigma_K}{\mu_K}\right)^2 + \frac{e^{-\alpha m_i}}{\mu_K}}$$

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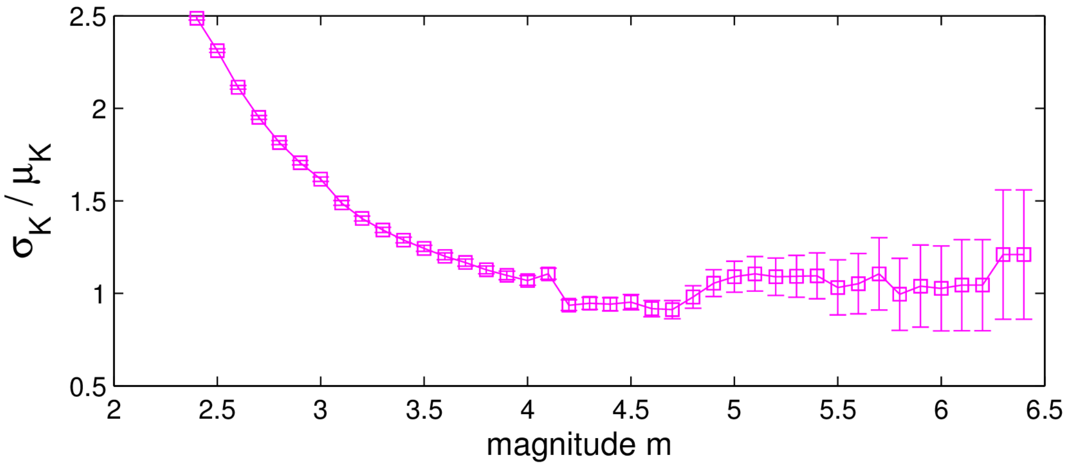

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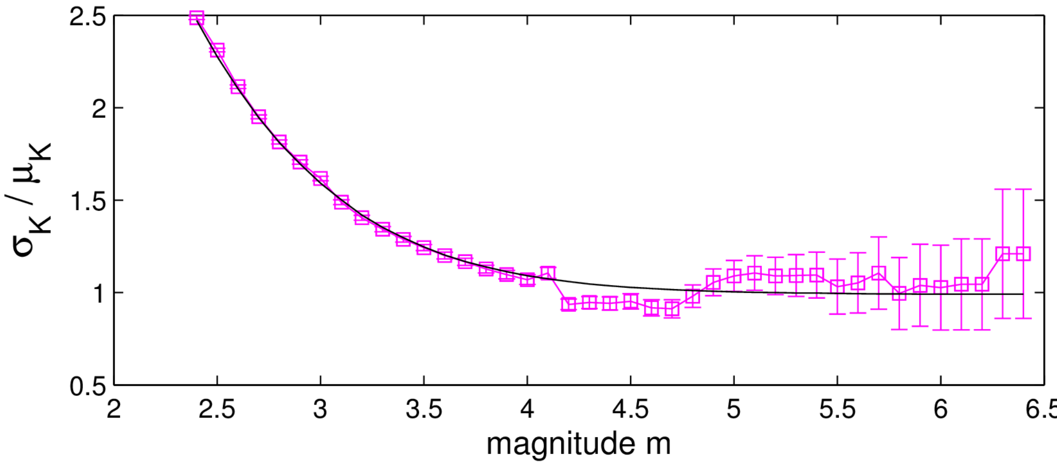
$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}}$ for earthquakes with magnitude ≥ m




**= 0.99 !**

$$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}} = \sqrt{\left(\frac{\sigma_K}{\mu_K}\right)^2 + \frac{e^{-\alpha m_i}}{\mu_K}}$$

# How to proceed when the cascade structure is unknown ?

**Simple procedure :**

- Determine pairs of parents / daughters using a model with variable K
- Number of aftershocks $n_i$ for each earthquake

- $n_i \sim \exp(\alpha m_i)$
- $K_i = n_i / \exp(\alpha m_i)$

- Compute standard deviation $\sigma(K_i \mid m_i \geq m)$
- Fit $\sigma(K_i \mid m_i \geq m)$ vs m curve to estimate $\sigma_K/\mu_K$.

- Determine pairs of parents / daughters using a model with variable K

**Start with a parameterized seismicity model (a guess)**

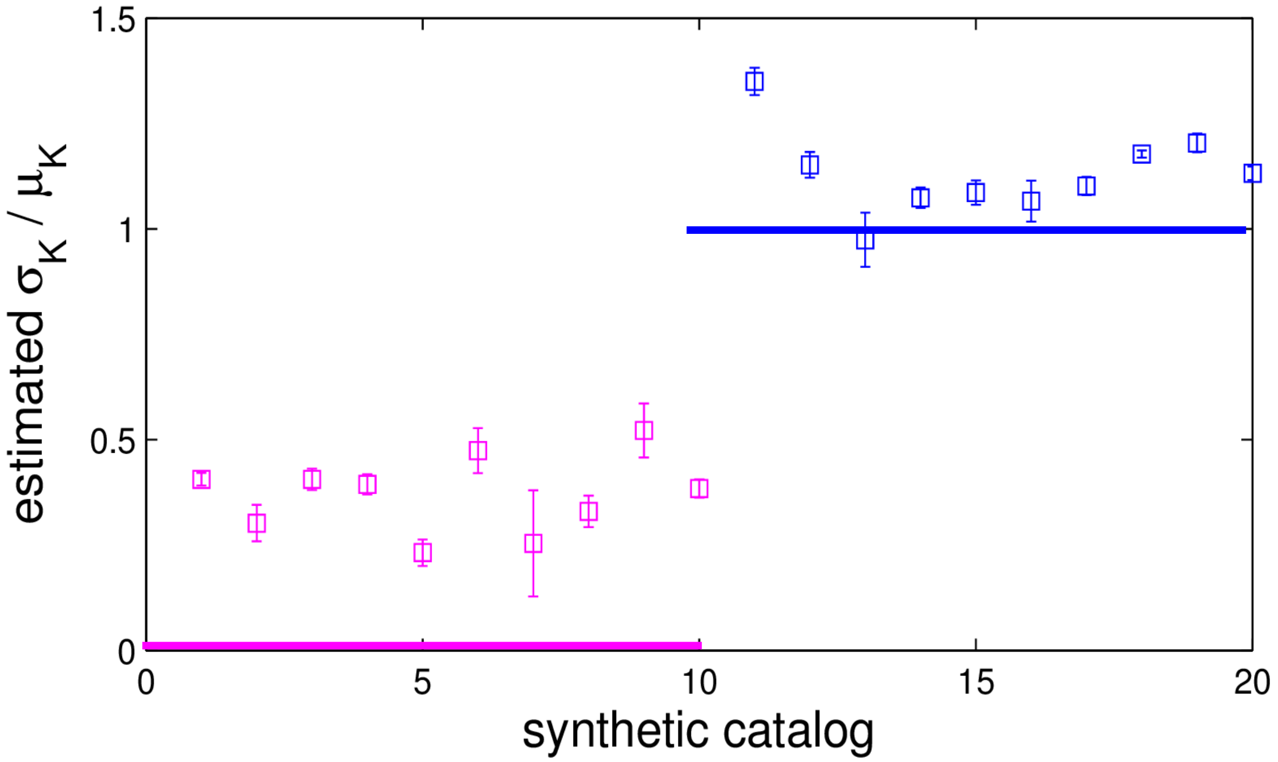
Draw a realization of a cascade structure

Compute marginal laws (Omori-Utsu law, productivity, spatial kernel) and fit parameters

Iterate

**Compute individual K values**

**Iterate**



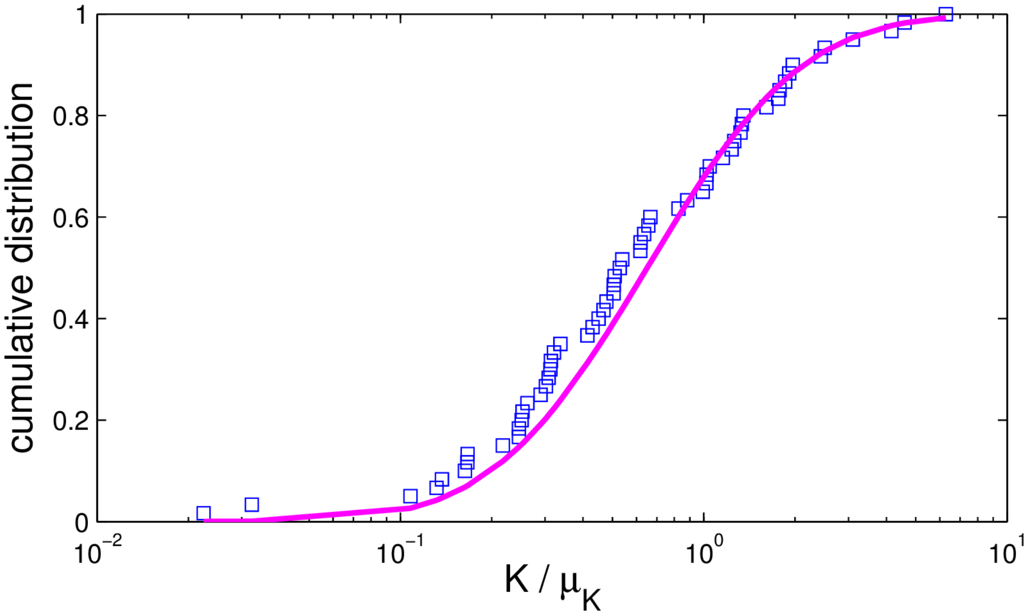

20 synthetic catalogs (10000 earthquakes) with or without variability in K
No model errors (analyzing model = generating model).

We analyze the dataset of Yang et al. (BSSA 2012) :

- 1981-2010 southern California
- 26816 earthquakes with m ≥ 2.4

$$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}}=1.05$$

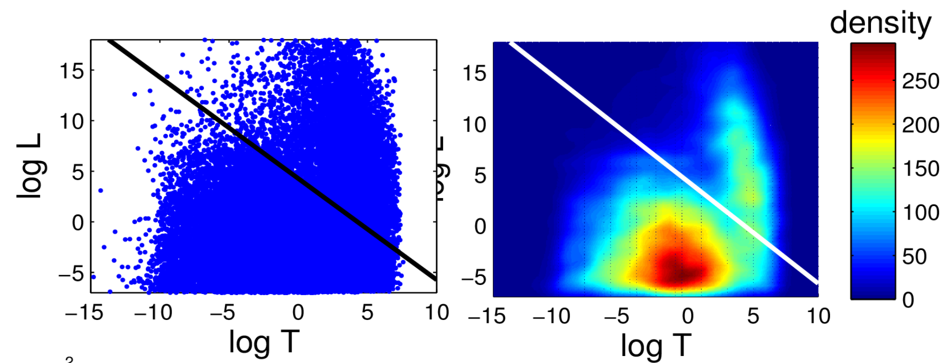
**M≥5.0 mainshocks**

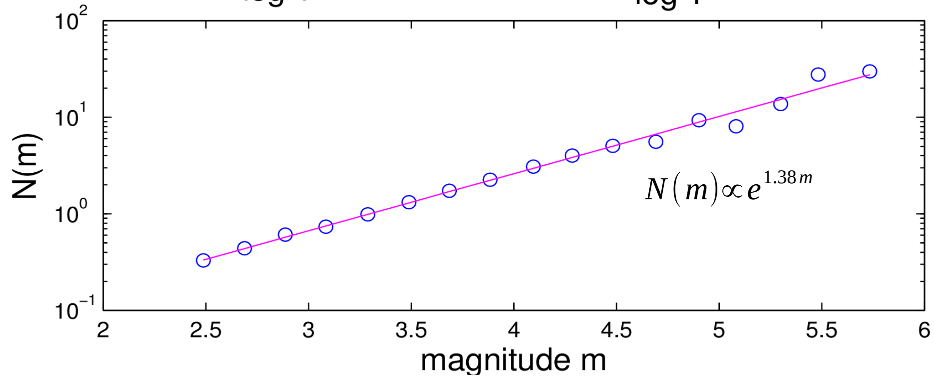


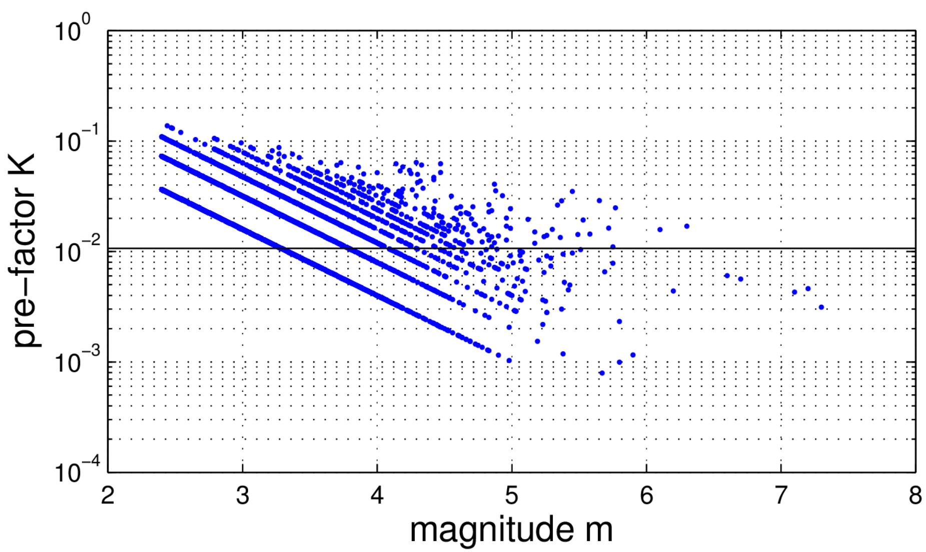

Log-normal law with $\sigma_{\log(\hat{K})}=0.92$

$$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}}=1.15 \quad for \quad m \geq 5$$

# Alternative analysis using an approach inspired by Zaliapin et al. (PRL 2008)



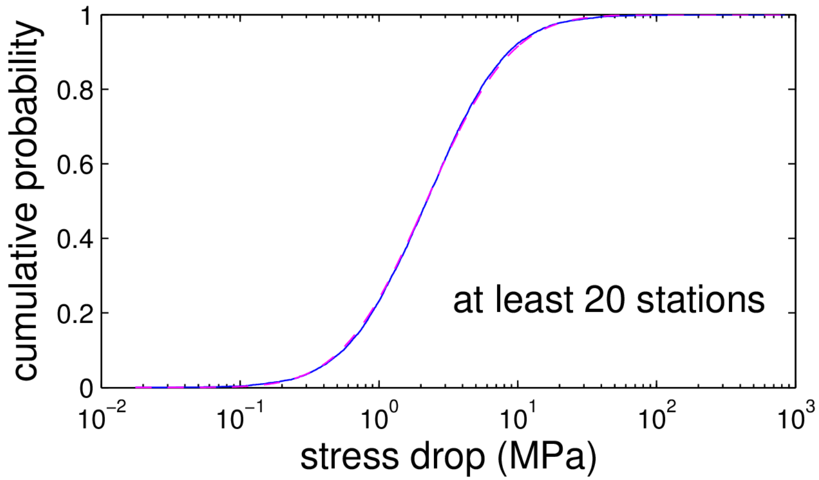




$$\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}} = 0.82$$

**Number of triggered aftershocks can depend on :**

- Local background rate ⟷ local stressing rate
- Depth
- Possible aseismic slip
- Rupture speed
- **Stress drop**

Stress drops of southern California earthquakes
$1.5 \leq m \leq 3.1$ earthquakes (Shearer et al., 2006)




$\Delta\sigma$ is **log-normal** with $\sigma_{\log(\Delta\sigma)} = 1.10$

| | | $\sigma_{\log(\Delta\sigma)}$ |
|---|---|---|
| Allmann & Shearer (JGR 2007) | Parkfield | 1.18 |
| Allmann & Shearer (JGR 2009) | global | 1.36 |
| Baltay et al. (GRL 2011) | Crustal Honshu | 0.97 |

**Without variability**

$$N = N_0 \; L^{\gamma} \propto e^{\alpha m}$$

$$L = \left( \frac{7 M_0}{16 \Delta \sigma} \right)^{1/3}$$

*Eshelby (1957)*

+

$$M_0 = 10^{1.5m + 9.1}$$

*Kanamori (1977)*

→

$$\gamma = \frac{2\alpha}{\log 10}$$

**With variability**

$$N = k \; \Delta\sigma \; L^{\gamma} \propto \underbrace{\Delta\sigma^{\delta}}_{K} \; e^{\alpha m}$$

$$L = \left( \frac{7 M_0}{16 \Delta \sigma} \right)^{1/3}$$

*Eshelby (1957)*

+

$$M_0 = 10^{1.5m + 9.1}$$

*Kanamori (1977)*

→

$$\gamma = \frac{2\alpha}{\log 10}$$

$$\delta = 1 - \frac{\gamma}{3}$$

$$K \propto \Delta\sigma^{1-2\alpha/3\log 10}$$

$$\alpha = 1.45 \Rightarrow K \propto \Delta\sigma^{0.58}$$

**Two consequences :**

- $\Delta\sigma$ log-normal ➡ K log-normal
- $\sigma_{\log(\Delta\sigma)} = 1.10$ ➡ $\sigma_{\log(K)} = 0.64$

compared to our estimated $\frac{\sigma_{\hat{K}}}{\mu_{\hat{K}}} = 1.05 \Rightarrow \sigma_{\log(K)} = 0.86$

- Triggering is **more variable** than a simple mean law
- Seismicity models with implemented K-variability are more realistic
- Dependence of K on physical (source) parameters needs to be studied
- **Estimation** of K-variability : not easy !
- Stress drop variability could explain most of the variability of K