MMax

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StatSei 2015 Potsdam

Determination of m_{max} is an important chalange in statistical seismology.
Available data comprises various time scales

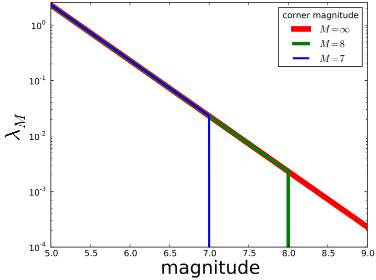
- catalogs ($< 500y$)
- paleo earthquakes ($< 100.000y$)
- geological fault displacement data ($> 100.000y$)

physical model of seismicity

Estimation is based on physical assumptions of earthquake distribution

- large events are independent
- occurence times are fully random (i.e. Poisson process)
- magnitudes follow a GR distribution up to some upper limit m_{max}

$$\lambda_M(m) \simeq 10^{-bm}, \quad m_0 \le m \le m_{max}$$



optimal estimation from catalogs

Likelihood function of n independent events

$$\begin{aligned} L(\{m_i\}|m_{max}) &= \lambda_M(m_1)\,\lambda_M(m_2)\ldots\lambda_M(m_n) \\ &= L_1(m_{max},\mu,n) \quad\times\quad L_2(b,\langle m\rangle) \end{aligned}$$

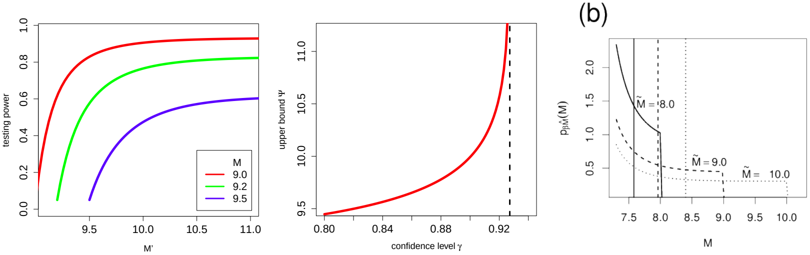
optimal observable

$$\mu = \max_{catalog}\{m_i\} = \text{maximum observed magnitude}$$

Only factor L_1 depends on m_{max}. $\Rightarrow$

All information about m_{max} is in μ and n
For geological data n is unknown.

Results for catalog data



- With high probability confidence interval $[\mu, \psi]$ is infinite
- Bayesian posterior reflects essentially the prior
- Bayesian posterior with flat prior in m_{max} is not normalizable
- Test for m_{max} does not have any power

Problems with catalog data

- Poor statistics, few events
- Frequency of large(st) events in time T of catalog is $\ll 1$
- Tail of distribution is poorly sampled

Geological data (total slip, paleo earthquakes) over time span T
uncertainty in n needs to be taken into account.

optimal observable

$$\mu = \max_T \{\text{magnitude}\}$$

Any other observable will produce less precise results
(e.g. larger confidence intervals)

Gumbel distribution

Extreme value distribution reads

$$\Pr(\mu \leq m | m_{\max}) = \sum_{n=0}^{\infty} \frac{\Lambda^n}{n!} e^{-\Lambda} \left(e^{-\beta m} - e^{-\beta m_{\max}} \right)^n$$

Gumbel distribution

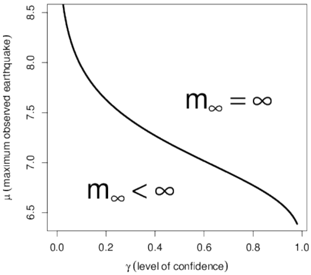
$$\Pr(\mu \leq m | m_{\max}) = \exp\left[-T\Lambda \left(e^{-\beta m} - e^{-\beta m_{\max}} \right) \right]$$

Optimal confidence interval

Confidence interval $[\mu, m_\infty]$ of level γ ($1-\gamma$ probability of failure)

$$m_\infty = -\frac{1}{b\log(10)}\log\left[10^{-b\mu} + \frac{b\log(10)}{T10^a}\log(1-\gamma)\right]$$

For reasonable levels of required confidence $m_\infty = \infty$
Example: $\mu = 6.7$ and $T = 45000 - 30000$ years

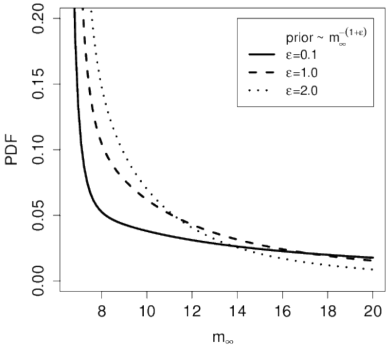


Bayesian posterior

The Bayesian posterior distribution of $m_{\max}$

$$p(m_{\max}|\mu) \propto 10^{\frac{T10^{a-bm_{\max}}}{b[\log(10)]^2}} p_0(m_{\max})$$

- Not normalisable for flat prior
- Posterior largerly determined by prior information



Proxy data

Maximal earhquake over time T is not available.
Proxy data may be used

- total slip over time T
- average slip rate over T
- total moment release over time T
- average moment rate over time T

BUT: confidence intervals based on these proxy data are larger than the ones based on μ

Seismic moment

Cumulated released seismic moment $\Leftrightarrow$ total slip over T
average moment rate $\Leftrightarrow$ average slip rate over T

Single event distribution of released moment depends on maximal moment $\mathcal{M}_{max}$

$$f_{\mathcal{M}_{max}}(\mathcal{M})$$

Sum of n events $\mathcal{M}_1 + \cdots + \mathcal{M}_n$ has distribution

$$f^{n(*)}_{\mathcal{M}_{\max}}(\mathcal{M}) \quad n\text{-fold convolution}$$

Total released moment during T (with cutoff for small moments)

$$L(\mathcal{M}|\mathcal{M}_{\max}, T) \propto \sum_{n=1}^{\infty} \frac{\Lambda^n e^{-\Lambda T}}{n!} f^{n(*)}_{\mathcal{M}_{\max}}(\mathcal{M}),$$

No explicit formula expression available. Relation to Levi process.
Sum is dominated by largest event $\Rightarrow$ proxy for μ

Problems

- High precision geodetic measurements of slip rates are of no use since time scale is too short
- Displacement over geological data comprises larger time scales, but still only a few seismic cycles are included
- Uncertainty in determination T
- Relation slip - moment release is not a fundamental law

Conclusion

- largest event μ provides best information about m_{max}
- confidence intervals are unbounded for reasonable confidence levels
- largest event may be replaced by cumulated moment/slip or moment/slip rate
- uncertainties on this proxy are bigger than the on based on μ
- cumulated moments from high precision geodetic observations are useless
- long term geological data of cumulated slip does neither contain enough seismic cycles

Maximum magnitude cannot be estimated from any source of data we are aware of