Statistical tests for the tail of the seismic-moment distribution of global shallow earthquakes

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Global CMT earthquake catalog

The data analyzed:
- Global CMT earthquake catalog
- We restrict to shallow events (depth < 70 km)
- Fixed minimum magnitude as 5.75
- From 1976 until 2014
- Sample size 6150
The size of earthquakes is measured either by...
- the magnitude: $m$
- the seismic moment, $M$

The relation if $M$ is measured in NewtonMetre units is given by

$$m = \frac{2}{3} \left( \log_{10} M - 9.1 \right)$$

GR law states that, for a given region, the magnitudes of earthquakes follow an exponential probability distribution (spatial dependence)

Equivalently...
...the seismic moment follows a power-law.

$$f(M) \propto \frac{1}{M^{1+\beta}}$$
The power-law model

**Branching process**

The power-law distribution has important physical implications, as it suggests an origin from a critical branching process or a self-organized-critical system.

**No finite moments**

The mean value $\langle M \rangle$ provided by the distribution turns out to be infinite. These elementary considerations imply that the GR law cannot be naively extended to arbitrarily large values of $M$, and one needs to introduce additional parameters to describe the tail of the distribution, coming presumably from finite-size effects.
The fit by Gutenberg-Richter law
Global catalog fitted by power-law

![Graph showing the fit of seismic moment to a power-law distribution.](image)

CMT catalog, 1977 - 2010, depth ≤ 70 km
power-law fit, exponent 1.68

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Tail distribution of the seismic-moment
Extreme value theory classifies the tails of distribution by a real number called tail index.

- If the decay of distribution is exponential, tail index is 0
- If the distribution is right-truncated, tail index is 1

**Tail test results**

- The 0-tail-index is not rejected over 8.3 (mag.)
- For the tail over 8.3, the test ”1-tail-index vs 0-tail index” concludes: reject 1-tail and to accept 0-tail
Kagan has enumerated the requirements that an extension of the GR law should fulfill.

**Tapered Gutenberg-Richter, Tap**

The survival function is $S_{tap}(M) \propto e^{-M/\theta / M^\beta}$

**The (left-) truncated gamma, TrG**

The density is $f_{trg}(M) \propto e^{-M/\theta / M^{1+\beta}}$.

Parameter $\theta$ represents a crossover value of seismic moment, signaling a transition from power law to exponential decay; so, $\theta$ gives the scale of the finite-size effects on the seismic moment.
Overview the models

For the power-law (PL) distribution (which yields the GR law for the distribution of $M \geq a > 0$) we have

$$f_{pl}(M; \beta) = \frac{\beta}{a} \left( \frac{a}{M} \right)^{1+\beta},$$

with $\beta > 0$. For the tapered Gutenberg-Richter,

$$f_{tap}(M; \beta, \theta) = \left[ \frac{\beta}{a} \left( \frac{a}{M} \right)^{1+\beta} + \frac{1}{\theta} \left( \frac{a}{M} \right)^{\beta} \right] e^{-(M-a)/\theta},$$

with $\beta > 0$ and $\theta > 0$. And for the left-truncated (and extended to $\beta > 0$) gamma distribution;

$$f_{trg}(M; \beta, \theta) = \frac{1}{\theta \Gamma(-\beta, a/\theta)} \left( \frac{\theta}{M} \right)^{1+\beta} e^{-M/\theta},$$

with $-\infty < \beta < \infty$ and $\theta > 0$, and with $\Gamma(\gamma, z) = \int_{z}^{\infty} x^{\gamma-1} e^{-x} dx$ the upper incomplete gamma function, defined for $z > 0$ when $\gamma < 0$. 
Maximum likelihood estimation of the parameters with their standard errors (s.e.) and maximum value of the log-likelihood function, \( l = \ln \hat{L} \), for the PL, Tap, and TrG distributions, using the whole dataset \( (N = 6150) \). The standard error for \( \hat{\beta} \) and \( \hat{\theta} \) is computed from the Fisher information matrix and corresponds to one standard deviation of the distribution of the parameter. The standard error for \( \hat{m}_c \) is computed from that of \( \hat{\theta} \) using the delta method.
Descriptive analysis of the fit

Comparison of the fits with the empirical distribution, using...

(upper curve, right axis) for the complementary cumulative distribution function

(lower curve, left axis) for the probability density function
Model selection approach

Maximum likelihood estimation and model selection tests based on the likelihood ratio.

No problem

When maximum likelihood is used under a wrong model, what one finds is the closest model to the true distribution in terms of the KL measure.

Problem

A big problem is that the change from power law to a faster decay seems to take place at the highest values of $M$ that have been observed, for which the statistics is very poor.
Likelihood comparision: two-to-two and year-by-year
In order to gain further insight, we simulate 1000 random samples following the truncated gamma distribution, with the parameters $\hat{\beta}_{trg}$ and $\hat{\theta}_{trg}$ obtained from ML estimation of the complete dataset, with the same truncation parameter $m_c = 5.75$ and number of points ($N = 6150$).

Reshuffle

In order that the conclusions do not depend on the time correlation, we reshuffle the simulated data in such a way that the occurrence of the rank statistics is the same as for the empirical data.

Finally, we repeat the simulations under Gutenberg-Richter model.
Comparison of the values of the estimated parameters of the TrG distribution, $\hat{\beta}_{trg}$ and $\hat{m}_{c, trg}$, for the empirical data and for 1000 simulations of the TrG distribution, using the final parameters.
1000 GR simulations

1000 TrG simulations
Some things to finish

Some bibliography...


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