Empirical Estimation of Fault Directionality for Improved Non-parametric Estimation of Branching Models for Earthquake Occurrences

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Branching Models

- **Self-Exciting Point Processes**
  A point process is self-exciting when the occurrence of an event increases the likelihood that another event will occur. In other words, the occurrence of an earthquake at \((t, x, y)\) triggers other earthquakes nearby.

- **Epidemic Type Aftershock Sequences (ETAS)**

  \[
  \lambda(t, x, y|\mathcal{H}_t) = \mu(x, y) + \sum_{i: t_i < t} \nu(t - t_i, x - x_i, y - y_i; m_i)
  \]

  \[
  = \mu(x, y) + \sum_{i: t_i < t} \kappa(m_i) \ g(t - t_i) \ f(x - x_i, y - y_i; m_i)
  \]

  \[
  = \mu(x, y) + \sum_{i: t_i < t} \frac{K_0}{((t - t_i) + c)^p} \cdot \frac{e^{\alpha(m - m_0)}}{((x - x_i)^2 + (y - y_i)^2 + d)^q}
  \]

  (Ogata 1998).
Extending ETAS

\[ \lambda(t, x, y|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} \nu(t - t_i, x - x_i, y - y_i; \omega_i, m_i) \]

\[ \lambda(t, x, y|\mathcal{H}_t) = \mu(x, y) + \sum_{i:t_i<t} \kappa(m_i) g(t - t_i) f(x - x_i, y - y_i; \omega_i) \]

- \( r_i \) = distance between mainshock epicenter \((x_i, y_i)\) and location \((x, y)\)
- \( \omega_i \) = angle to \((x, y)\) relative to fault plane of \((x_i, y_i)\)
Spatial Distribution of Aftershocks

- Many ETAS parameterizations assume an isotropic spatial distribution of aftershocks.
- Aftershocks are found to lie preferentially along mainshock fault planes in Southern California seismological data.
- Various investigations have taken place to inform an aftershock’s anisotropic spatial distribution.
- Parametric and semi-parametric models have been proposed such as a Normal model in Ogata 1998 and a Tapered Pareto in Wong et al. 2009.
- Seismic moment tensor information can be used to estimate directionality of rupture for selected mainshocks.
Fault Directionality

Using the following steps we can estimate a fault direction for each mainshock and estimate angular separation based on prior seismicity.

1. Identify all $M_1 \geq c_1$ events within some area surrounding a $M_2 > c_2$ event, where $c_1 < c_2$. Here $c$ is a magnitude cuttoff and $M$ is the magnitude of an earthquake.

2. Calculate the Euclidean distance between a $M_2$ earthquakes epicenter and all other $M_1$ events denoted $r_{ij}$.

   \[ r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]

3. Fit a weighted ordinary least squares regression constrained to pass through the $M_2$ epicenter with weights $w_i = 1/r_{ij}$.

4. Store the coefficient of the weighted regression $\phi_i$ as the estimated fault plane for each $M_2$ event.
Weighted Least Squares

In the model:

\[ Y_i = \beta X_i + \epsilon_i \]
\[ \text{Latitude}_i = \beta \text{Longitude}_i + \epsilon_i, \]

where \( \epsilon_i \sim N \left( 0, \frac{\sigma^2_i}{r_i} \right) \).

Instead of minimizing

\[ RSS(\beta) = \sum_{i=1}^{n} (Y_i - \beta X_i)^2 \]

we can minimize the weighed summed of squares

\[ WSS(\beta, w) = \sum_{i=1}^{n} w_i (Y_i - \beta X_i)^2 \]

\( w_i \) are the weights. Ordinary least squares is a special case where all the weights \( w_i = 1 \).
Angular Separation

• In order for the procedure to be completely objective, we do not distinguish subjectively between mainshocks and aftershocks.

• Earthquake $i$ comes first and is called the **mainshock** and $j$ comes after and is this the **aftershock**.

• $\omega_{ij}$ is the angle made by the segment connecting mainshock epicenter $(x_i, y_i)$ to aftershock epicenter $(x_j, y_j)$, and the line through $(x_i, y_i)$ with direction $\phi_i$ for each pair of earthquakes where $t_i > t_j$.

\[
\omega_{ij} = \arctan \left( \frac{|m_1 - m_2|}{1 + m_1 \cdot m_2} \right),
\]

where $m_1$ is the slope of the line passing through $(x_i, y_i)$ and $(x_j, y_j)$. $m_2$ is estimated the fault direction $\phi_i$.

• As in Wong et al. 2009, we do not distinguish the two zones $\omega_{ij} \in \left[ 0, \frac{\pi}{2} \right]$ and $\omega_{ij} \in \left[ \frac{\pi}{2}, \pi \right]$. 
Non-parametric Estimation of ETAS Triggering Function

Utilizing “Model Independent Stochastic Declustering” (MISD), introduced by Marsan and Lengliné (2008), non-parametrically estimate the triggering function of the conditional intensity.

\[
\hat{f}(r, \omega)_{k,\ell} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{i-1} C_{k,\ell} p_{ij}}{A_{k,\ell} \sum_{i=1}^{N} \sum_{j=1}^{i-1} p_{ij}}
\]

\[
\begin{align*}
C_{k,\ell} &= \left\{(i, j) \middle| \delta r_k \leq r_{ij} \leq \delta r_{k+1}, \delta \omega_{\ell} \leq \omega_{ij} \leq \omega_{\ell+1}, i > j \right\} \\
A_{k,\ell} &= \frac{\delta \omega_{\ell}}{2} \left( r_{k+1}^2 - r_k^2 \right)
\end{align*}
\]
Data

- Latitude [32, 44]
- Longitude [−126, −114]
- Jan 1, 1980 to Jun 1, 2015
- 204,758 \( M_1 \geq 2.0 \) events
- 677 \( M_2 > 4.5 \) events
- Depth \( \leq 75 \text{km} \)
Fault Estimates

Figure: Fault planes with nearly linear seismicity
Fault Estimates

Figure: Fault planes with a poor fit to nearby seismicity
Fault Estimates

**Figure**: Fault planes with nearly linear seismicity but a problematic estimate
Angular Separation Examples

Angular separation (\(\omega\)) between fault plane (red) and aftershock (blue), centered at location of mainshock.

- **30 km 11 degrees**
- **6 km 16 degrees**
- **31 km 46 degrees**
- **55 km 34 degrees**
Angular Separation

Angular separation ($\omega$) between fault plane and aftershock, centered at location of mainshock and arranged according to distance $r$ between mainshocks and aftershocks, for all faults.

- $0km < r \leq 5km$
- $5km < r \leq 10km$
- $10km < r \leq 20km$
- $20km < r \leq 50km$
Angular separation ($\omega$) between fault plane and aftershock, centered at location of mainshock, for better fitting faults.

- $0\,km < r \leq 5\,km$
- $5\,km < r \leq 10\,km$
- $10\,km < r \leq 20\,km$
- $20\,km < r \leq 50\,km$
Conclusion

- Alternative statistical models for fault estimation
  - Deming Regression
  - Robust Regression

- Alternative criteria for selection of $M_1$ events used in fault estimation
  - Dynamic Spatial Windows
  - Nearest Neighbors

- Hybrid approach using physical and statistical models
  - Best rated focal mechanisms
  - Subset to certain types of faults

- Questions?
References


