StatSei9

On the convolution of stochastic processes for modelling strong earthquake occurrences: a multi-rupture model driven by a self-correcting model

R. Rotondi and **E. Varini**, *CNR - Istituto di Matematica Applicata e Tecnologie Informatiche, Milano, Italy*

INTRODUCTION

Two features of earthquake generation are widely noted:

- earthquakes tend to occur in clusters (Fig.1), sometimes referred to swarms, foreshocks and aftershocks
  $\longrightarrow$ behaviour described by ***self-exciting models***.
- the fault ruptures that generate earthquakes decrease the amount of strain present at the locations along the fault where rupture occurs
  $\longrightarrow$ behaviour described by ***self-correcting models***.

Stochastic models that try to capture both these diametrically opposed features should reconcile contrasting trends.

We consider a sequence of earthquakes whose occurrence time and magnitude are observed. Among these events we distinguish the ones whose magnitude exceeds a threshold magnitude m_f and we name them **leaders**; the remaining events are called **subordinates**.

The cluster behaviour of subordinates around leaders is modelled through a stochastic process with a bathtub-shaped hazard function.

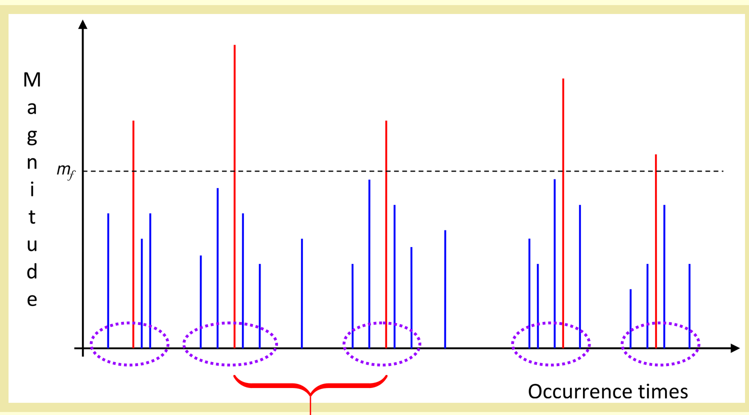


Figure 1
Magnitude vs occurrence time.
– – – : threshold magnitude m_f;
leaders : events of magnitude at least m_f;
subordinates : events of magnitude less than m_f;
········· : clusters of earthquakes.

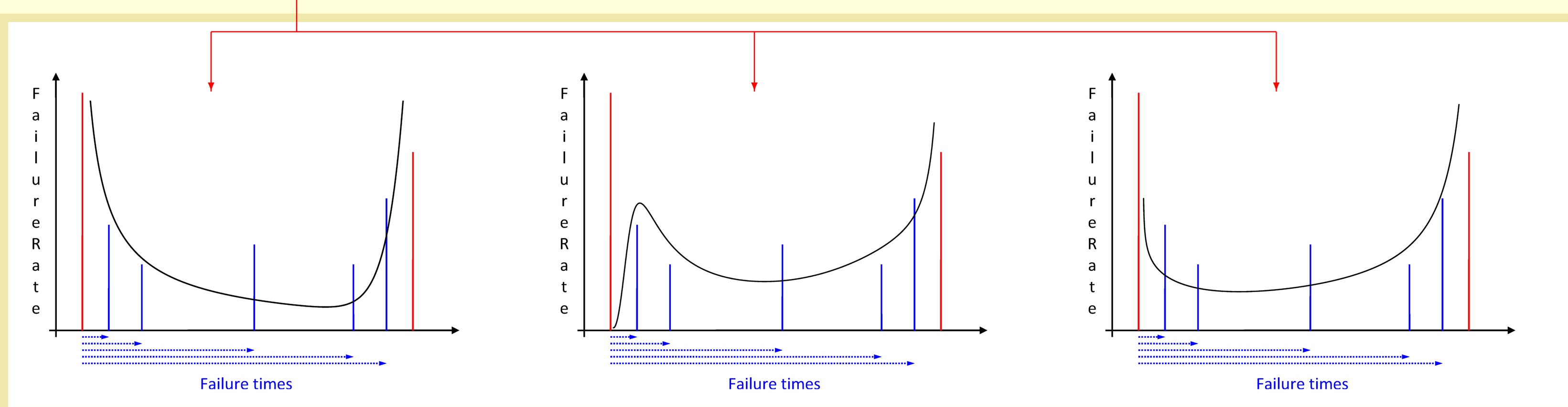


Figure 2: Examples of bathtub-shaped hazard function of Generalized Weibull distributions: *additive Weibull* (left), *extended Weibull* (middle), and mixture of Weibull (right).

PROBABILITY MODEL

Occurrence times are modelled through the superposition of stochastic processes under the assumption of separability with respect to magnitude.

Leaders: stress release model

Let $\{(t_i, m_i):\ m_i \ge m_f,\ i = 1, ..., n\}$ be the set of the leaders. Leaders follow a stress release model, that is a self-correcting point process defined by the hazard function:

$$\lambda(t) = exp\left\{\alpha + \beta\left[\rho t - \sum_{i:t_i<t} x_i\right]\right\},$$

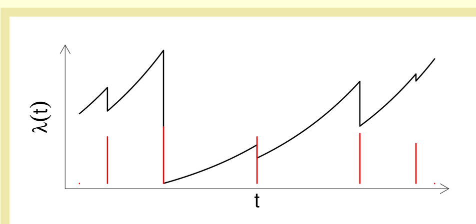


Figure 3: Hazard function of the stress release model.

where α, β and ρ are parameters and x_i denotes the scaled energy released by the i-th leader [3], that is the ratio between seismic energy E_s and seismic moment M_0.
By expressing E_s according to Senatorski (2007) and M_0 according to Kanamori and Brodsky (2004), each x_i is given by:

$$x_i = \frac{E_s}{M_0} \propto \frac{M_0^{1.5}/\sqrt{A_i}}{M_0} = \sqrt{\frac{M_0}{A_i}},$$

where A_i is obtained from the Wells-Coppersmith's regression models (1994) for different focal mechanisms.

Subordinates: generalized Weibull model

For each $i = 1, ..., n-1$, let $\{(\tau_{ij}, m_{ij}):\ t_i < \tau_{ij} < t_{i+1},\ m_0 \le m_{ij} < m_f,\ j = 1, ..., n_i\}$ be the set of the subordinates.
Each time τ_{ij} is normalized on $(0, 1)$ so as to account for the different length of the interval between consecutive leaders to which it belongs:

$$s_{ij} = \frac{\tau_{ij} - t_i}{t_{i+1} - t_i}.$$

The normalized times are assumed to be realizations of order statistics of a distribution truncated on $(0, 1)$. So far we considered the following generalized Weibull distributions [1], whose hazard functions $h_W(s)$ may have a bathtub shape (Fig.2):

Modified Weibull: A three-parameter generalized Weibull model.

$$h_{W_m}(s) = b\,(a + cs)\ s^{a-1}\ e^{cs}$$

Flexible Weibull: Simple and yet very flexible model applied to reliability data.

$$h_{W_f}(s) = exp\left\{as - \frac{b}{s}\right\}\left(a + \frac{b}{s^2}\right)$$

Magnitude: exponential model

According to the Gutenberg-Richter law, magnitude m is exponentially distributed with rate b and support $[m_0, +\infty)$. Its density function is :

$$f(m) = be^{-b(m-m_0)}$$

The number n_i of subordinates in each interval between consecutive leaders, comes out to have a Geometric distribution:

$$P(n_i) = p(1-p)^{n_i},$$

where $p = \bar{F}(m_f) = 1 - F(m_f)$ is the probability of exceeding magnitude m_f [2]. Different magnitude distributions can be also considered, e.g., the Pareto model or the tapered Pareto model.

LIKELIHOOD FUNCTION AND ESTIMATION METHOD

Conditioning the subordinates on the leader-events allows us to write the **likelihood function**:

$$\mathcal{L}(data \mid \theta) = P(leaders,\ subordinates \mid \theta) = \tag{1}$$
$$= \prod_{i=1}^{n} \lambda(t_i)\ exp\left\{-\int_{t_1}^{t_n} \lambda(u)du\right\} \times \prod_{i=1}^{n} \frac{f(m_i)}{\bar{F}(m_f)} \times$$
$$\times \prod_{i=1}^{n-1} \bar{F}(m_f)\,[F(m_f)]^{n_i} \times \prod_{i=1}^{n-1}\left[n_i! \prod_{j=1}^{n_i} \frac{f_W(s_{ij})}{F_W(1)}\right] \times \prod_{i=1}^{n-1}\prod_{j=1}^{n_i} \frac{f(m_{ij})}{F(m_f)}$$

Metropolis-Hastings algorithm

step 1 : let $p(\theta)$ be the prior distribution of the parameter vector,
step 2 : select θ_0 from $p(\theta)$
step 3 : for all $i = 1, ..., K$,
- select $\tilde{\theta}$ from the proposal distribution $q(\theta \mid \theta_{i-1})$,
- evaluate $\alpha(\theta_{i-1}, \tilde{\theta}) = max\left(1, \frac{p(\tilde{\theta})\mathcal{L}(data \mid \tilde{\theta})q(\theta_{i-1} \mid \tilde{\theta})}{p(\theta_{i-1})\mathcal{L}(data \mid \theta_{i-1})q(\tilde{\theta} \mid \theta_{i-1})}\right)$,
- set $\theta_i = \tilde{\theta}$ with probability $\alpha(\theta_{i-1}, \tilde{\theta})$, set $\theta_i = \theta_{i-1}$ otherwise.

Bayesian estimation

The posterior distribution $\pi(\theta)$ is approximated by a Markov chain $\theta_1,\ \theta_2,\ ...,\ \theta_K$ of parameter vectors generated by a *Metropolis-Hastings algorithm.*

APPLICATION

DATA

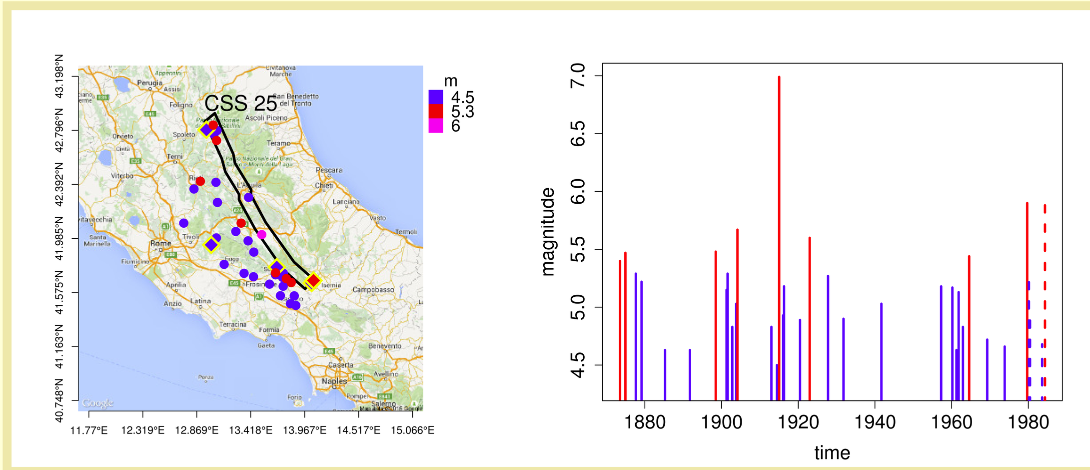
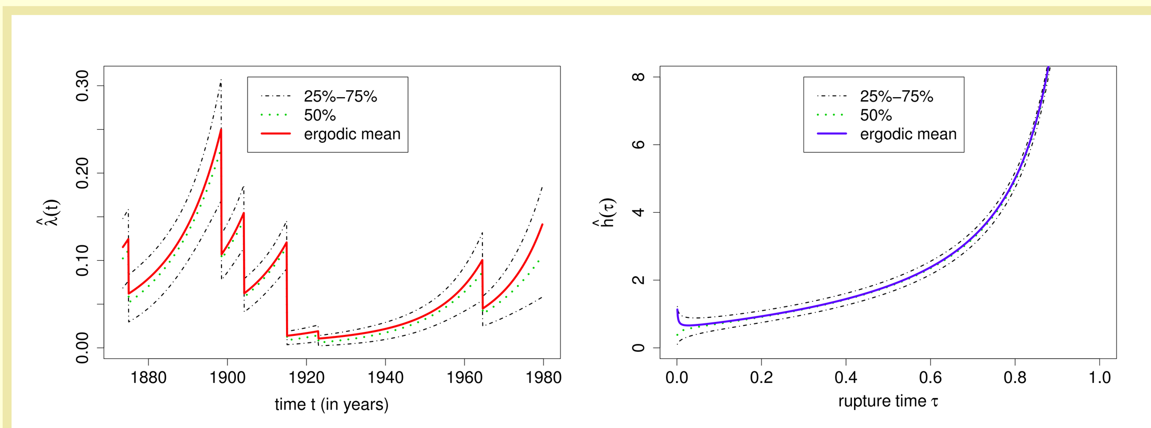


Figure 4: Events drawn from CPTI04 catalog and associated with the composite seismogenic source 25 of the Italian Database DISS (version 3.0.2).

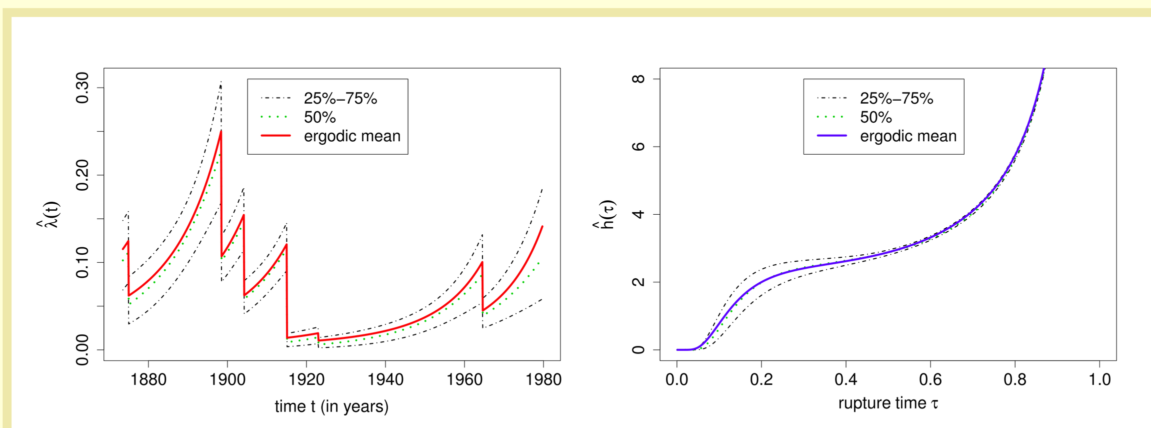
(right)
Solid lines: events for parameter estimation in [1873, 1980] ,
Dashed lines: events for model scoring in [1980, 1984].

LIKELIHOOD



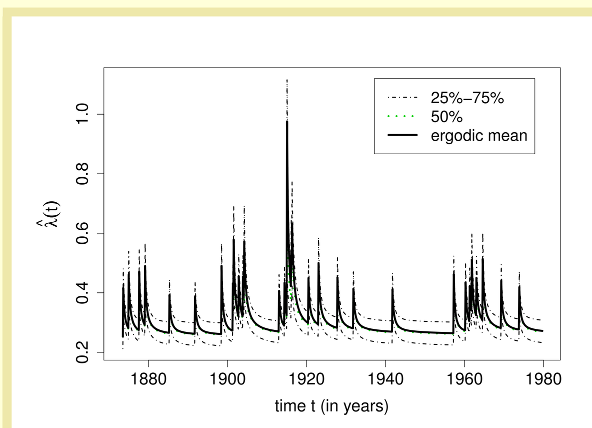
stress relese model + modified Weibull model

Marginal $Log\mathcal{L}$ = **-3.34**



stress relese model + flexible Weibull model

Marginal $Log\mathcal{L}$ = **-4.32**



ETAS model

Marginal $Log\mathcal{L}$ = **-30.79**

SCORING

We evaluate the log-likelihood of the data in the time interval [1980,1984].

	stress release + modified Weibull	stress release + flexible Weibull	ETAS
$Log\mathcal{L}$	-2.34	-2.52	-5.60

References

[1] Lai C.D. (2014) *Generalized Weibull distributions* Springer
[2] Kagan Y.Y. (2014), *Earthquakes. Models, statistics, testable forecasts* Wiley
[3] Varini E., Rotondi R. (2015), *Probability distribution of the waiting time in the stress release model: the Gompertz distribution Environ Ecol Stat*, DOI 10.1007/s10651-014-0307-2 (online first).