Spatial Variation on Earthquake Recurrence Time Distribution in Japan

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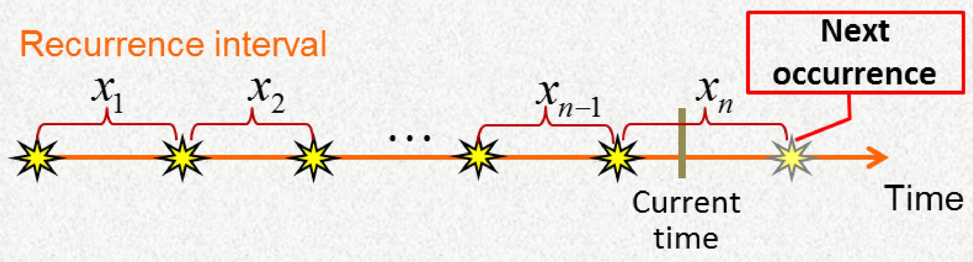
Abstract

We propose an extended model of the renewal process with Brownian Passage Time distribution BPT(μ,α) whose coefficient of variation (CoV) α has spatial trends. We show that the spatial distribution of CoV α tends to be larger in the region where the density of active faults is higher. We compare the probability forecasts by our proposed model with those by the Headquarters for Earthquake Research Promotion (HERP) in Japan.

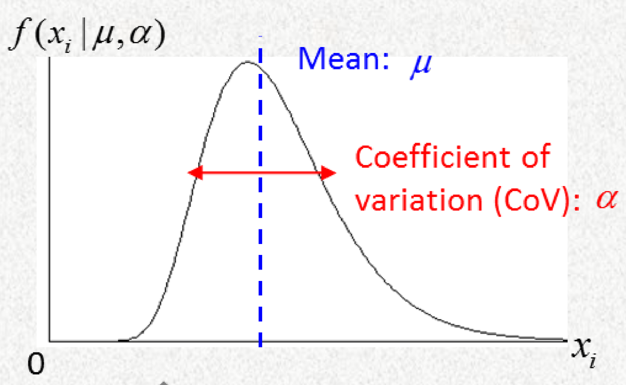
Base model

- Renewal process using the Brownian Passage Time (BPT) distribution



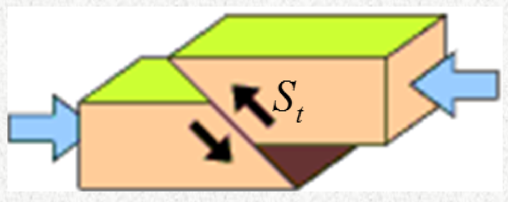
$$x_i \overset{i.i.d.}{\sim} f(x_i \mid \mu,\alpha) = \sqrt{\frac{\mu}{2\pi\alpha^2 x_i^3}} \exp\left\{-\frac{(x_i-\mu)^2}{2\mu\alpha^2 x_i}\right\}$$

probability density for recurrence intervals



$$\mu = \frac{\Delta S}{\lambda} \qquad \alpha = \frac{\sigma}{\sqrt{\lambda \Delta S}}$$

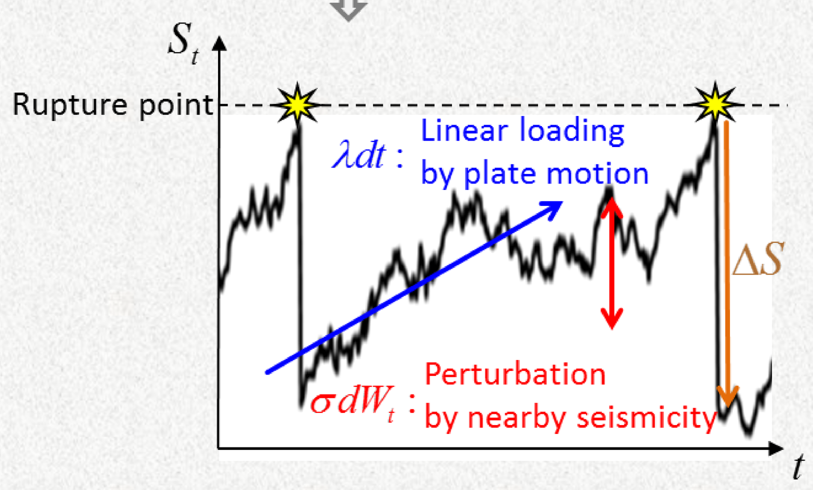
- Cf) Background model



$dS_t = \lambda dt + \sigma dW_t$

S_t : Coulomb failure stress

W_t : Standard Brownian motion



Parameter inference and forecast by HERP Japan

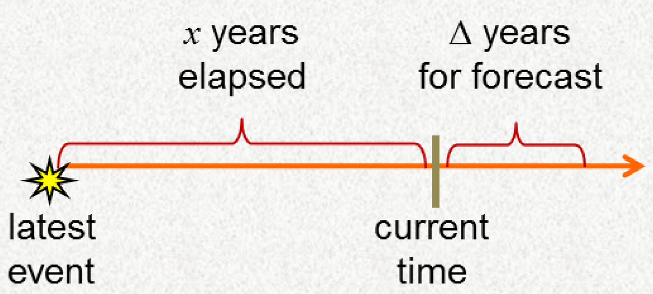
Mean recurrence time $\hat{\mu} = \begin{cases} \text{(1) Mean of past recurrence intervals} \\ \text{(2) Slip size per event / long-term slip rate} \end{cases}$

Coefficient of variation (CoV) $\hat{\alpha} = 0.24$ (Common estimate for 4 active faults and applied for all active faults in Japan)

Probability of occurrence in Δ years after x years elapsed from the latest event:

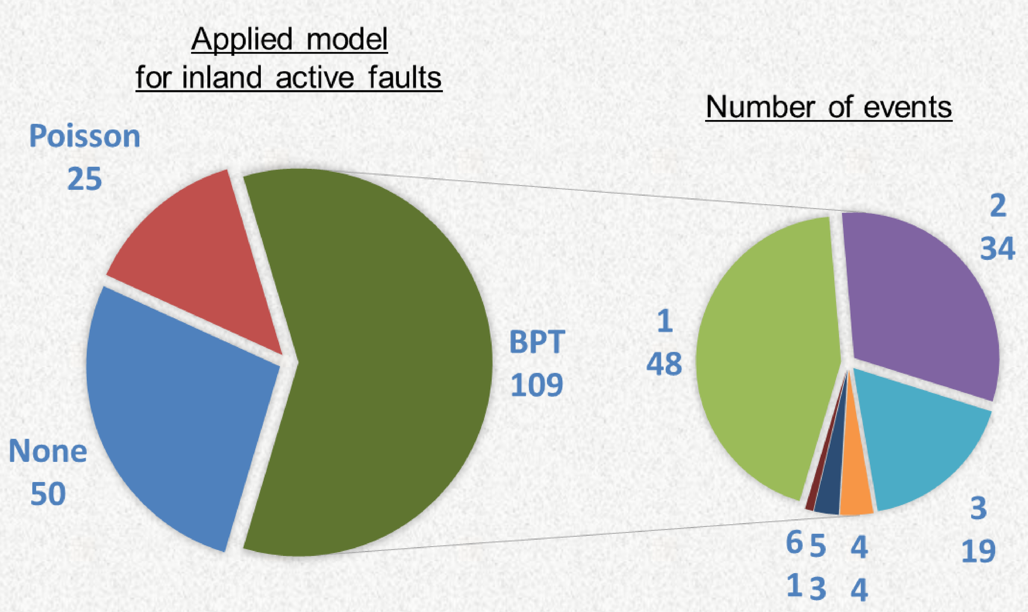
$$F_x(\Delta) = \int_x^{x+\Delta} f(y \mid \hat{\mu},\hat{\alpha})dy \Big/ \int_x^{\infty} f(y \mid \hat{\mu},\hat{\alpha})dy$$

probability that the recurrence interval will be between x and $x+\Delta$ yrs; probability that the recurrence interval will be over x yrs

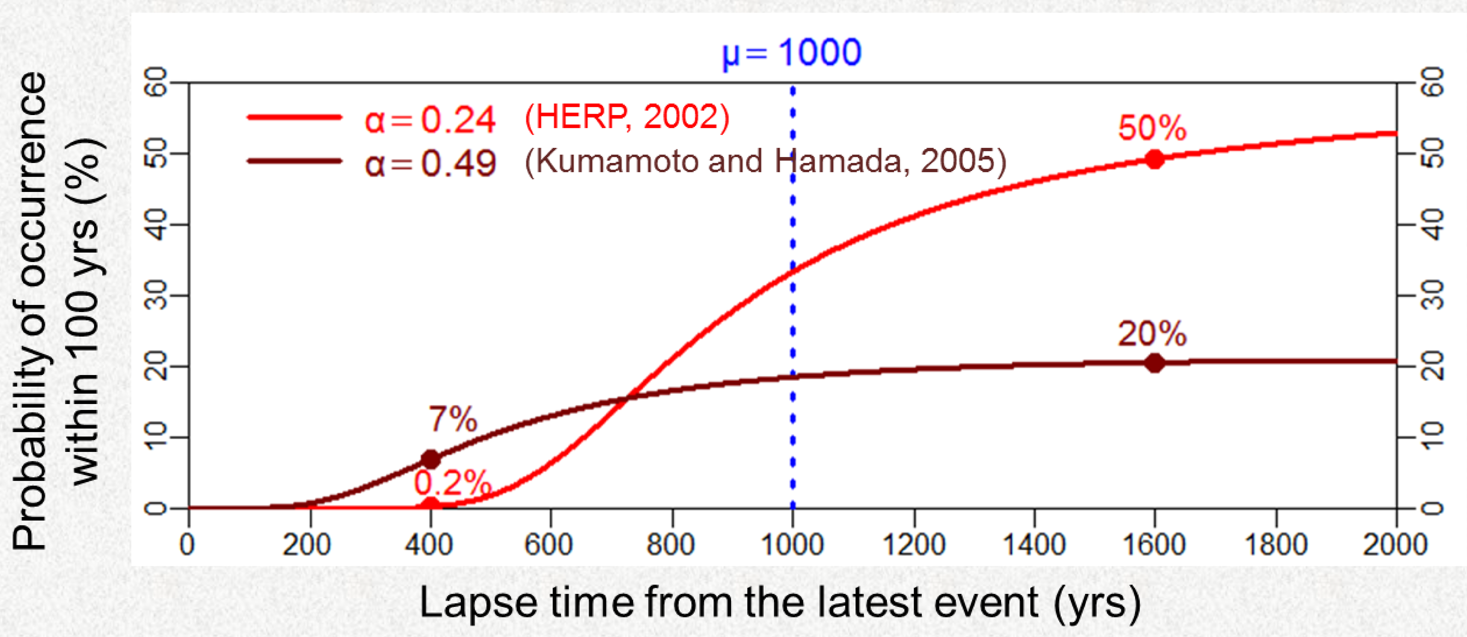


Influence of CoV's estimate on forecast

- Numbers of events are not enough to estimate reliable CoV α for most active faults in Japan.



- Various CoV estimates are given and make significant difference in forecast.

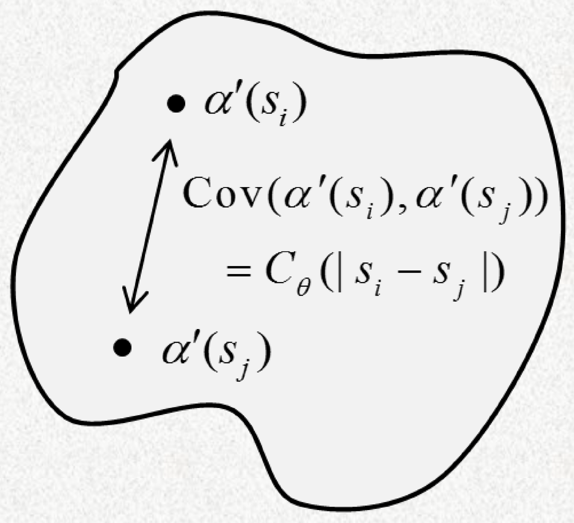


Spatial distribution for CoV α

- Prior distribution for coefficient of variation α

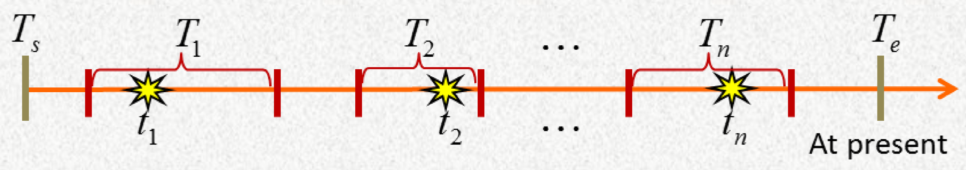
$$(\alpha'_1,\cdots,\alpha'_m) = \log(\alpha_1,\cdots,\alpha_m) \sim N\left(\theta_0 \mathbf{1}_m, \Sigma\right)$$

- Location vectors of m faults: $s_1,\dots,s_m$
- Covariance matrix: $\Sigma_{ij} = C_\theta(s_i - s_j)$
- Covariance function: $C_\theta(s) = \theta_1^2 \exp\left\{-\theta_2 |s|^2\right\}$
- Predictor of α' at arbitrary location s_{new}:



$$E\left[\alpha'(s_{new})\right] = \sum_{i=1}^m \sum_{j=1}^m C_\theta(s_{new} - s_i)\Sigma_{ij}^{-1}\alpha'(s_j)$$

- Occurrence date of past earthquakes are often uncertain and given in ranges.



$$L(\mu,\alpha \mid T_1,\cdots,T_n) = \int_{t_1\in T_1}\cdots\int_{t_n\in T_n} L(\mu,\alpha \mid t_1,\cdots,t_n)dt_1\cdots dt_n$$

$$L(\mu,\alpha \mid t_1,\cdots,t_n) = \int_{-\infty}^{T_s} f(t_1 - t \mid \mu,\alpha)\mu^{-1}dt \prod_{i=1}^{n-1} f(t_i - t_{i-1} \mid \mu,\alpha)\int_{T_e}^{\infty} f(t - t_n \mid \mu,\alpha)dt$$

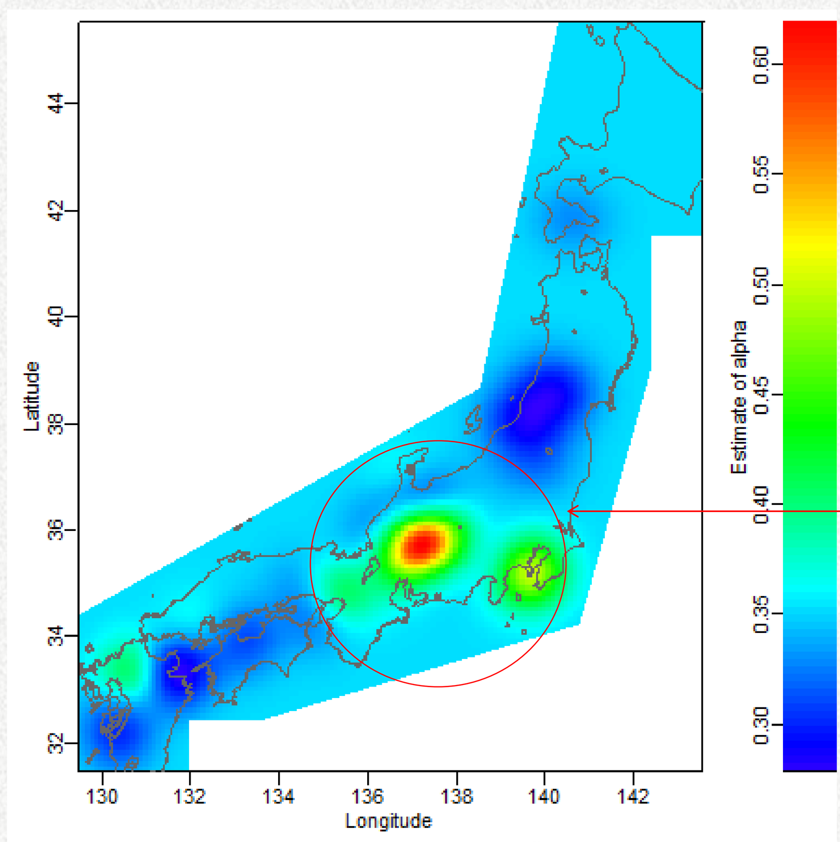
- Hyper-parameter θ is estimated by empirical Bayes method.

$$\hat{\theta} = \arg\max_\theta L(\theta),$$

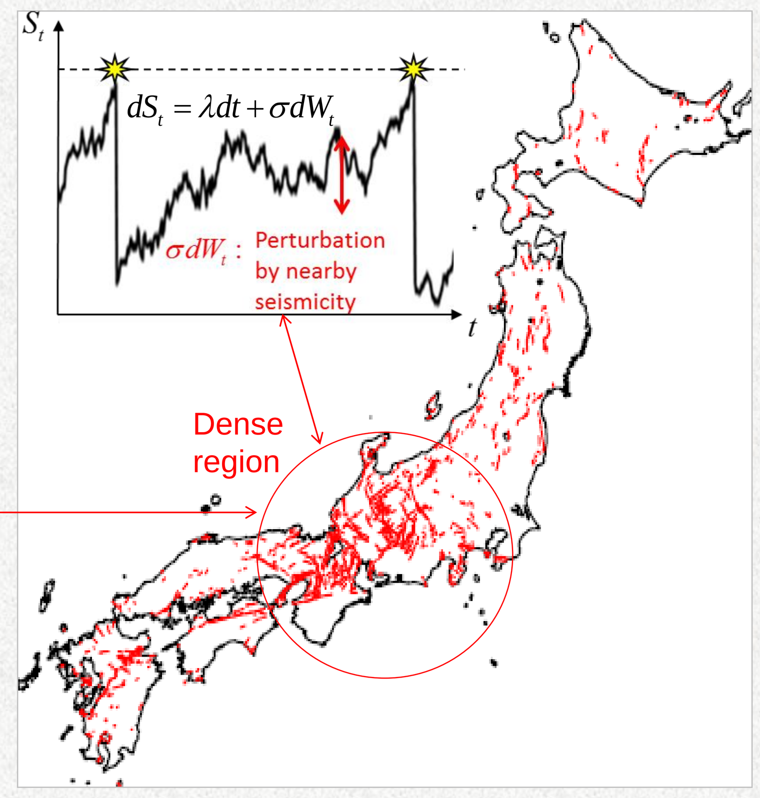
$$L(\theta) = \iint \pi(\mu_1,\cdots\mu_m)\pi(\alpha_1,\cdots\alpha_m \mid \theta)\prod_{i=1}^m L(\mu_i,\alpha_i \mid T_{i,1},\cdots,T_{i,n_i})d\mu_i d\alpha_i$$

$$\pi(\mu_1,\cdots,\mu_m) \propto \prod_{i=1}^m \frac{1}{\mu_i} \qquad (100\text{yrs} \le \mu_1,\cdots,\mu_m \le 1000000\text{yrs})$$

- Estimated spatial distribution of α's predictor ranges from 0.28 to 0.62.



Spatial distribution for CoV α



Map of active faults

Our forecasts and comparison with HERP

- Our forecasts for the next 30 yrs with the MAP estimates of μ and α are less variant than HERP's forecasts.